

Introduction

李宏毅

Hung-yi Lee

Course Information

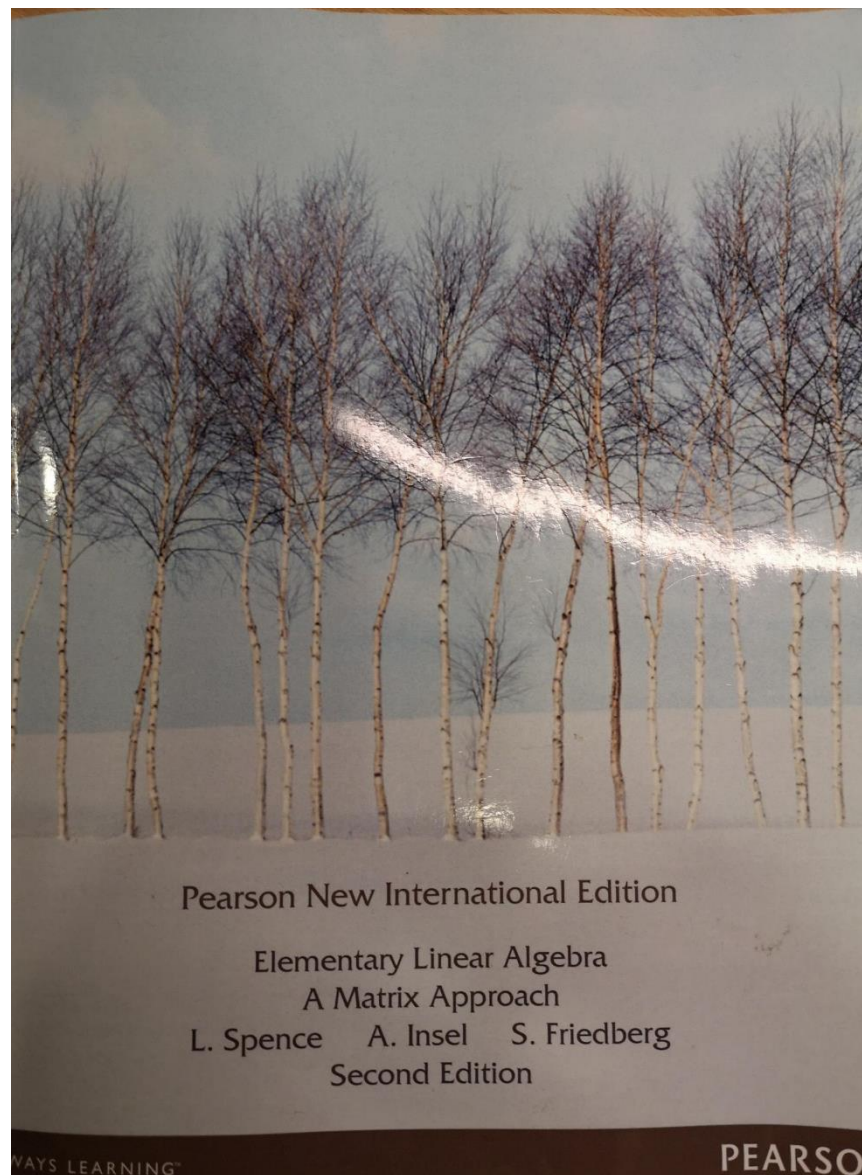
課程資訊

- 上課時間：
 - 週三上午 9:10 – 10:00
 - 週五上午 10:20 – 12:10
- 上課地點：博理112
- 評量方式：
 - 期中考 (35%)
 - 期末考 (35%)
 - 六次作業 (每個作業 6%)
 - 取最高的五次計分

教科書

- Elementary Linear Algebra - A Matrix Approach, **2nd Ed.**, by L. E. Spence, A. J. Insel and S. H. Friedberg

上課不完全按照教科書內容，
但考試範圍都會講到



考試範圍

- 考試範圍：教科書所有未打星號* 章節

5.5* APPLICATIONS OF EIGENVALUES

In this section, we discuss four applications involving eigenvalues.

MARKOV CHAINS

Markov chains have been used to analyze situations as diverse as land use in Canada [3], economic development in New Zealand [6], and the game of chance [1] and [2]. This concept is named after the Russian mathematician Andrey Markov (1856–1922), who developed the fundamentals of the theory at the beginning of the twentieth century.

A **Markov chain** is a process that consists of a finite number of states and transition probabilities p_{ij} , where p_{ij} represents the probability of moving from state j to state i . Note that this probability depends only on the present state j and not on the past. The movement of population between the city and suburbs described in Section 2.1 is an example of a Markov chain with two states (city and suburbs).

In Section 5.3, we investigate matrices that are similar to a diagonal matrix.

COMPLEX EIGENVALUES*

We have seen in Example 3 that not all $n \times n$ matrices or linear operators on \mathbb{R}^n have real eigenvalues and eigenvectors. The characteristic polynomial of such a matrix

* The remainder of this section is used only in the description of harmonic motion (an optional topic in Section 5.5).

期中、期末考

- 期中和期末考則採各班統一時間舉行及命題
- 時間為期中考週和期末考週的週五
- 考試規則：
 - 除非生病(需醫師開立證明)，不得請假及要求補考
 - 考試作弊者，學期成績不及格
 - 不接受以任何方式求情要求加分或使學期成績及格

作業

- 作業不是勾課本習題 (課本習題請自行練習)
- 作業是線性代數的應用
- 需要使用 python
- 作業繳交日期和方式等細節另行公告

其他注意事項

- 上課投影片會放在李宏毅的個人網頁上
 - http://speech.ee.ntu.edu.tw/~tlkagk/courses_LA19.html
- FB 社團：NTUEE Linear Algebra (2019)
 - <https://www.facebook.com/groups/667575690377769/>
 - 歡迎討論任何和線性代數相關的問題
 - 請務必加入FB社團，重要訊息除了透過學校 ceiba 寄 e-mail 外，也會透過FB 社團公告
- 助教信箱：LinearAlgebraLee@gmail.com
- 有關加簽

What are
we going to learn?

System

- A system has input and output (function, transformation, operator)

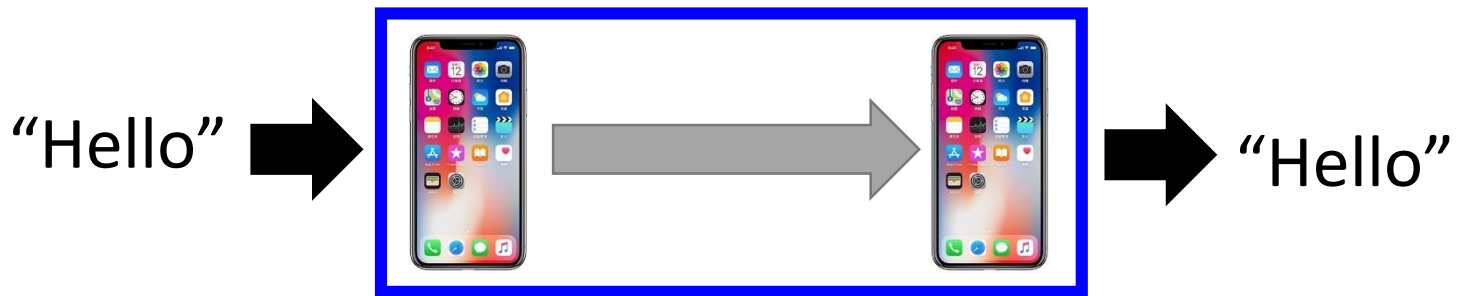
Speech Recognition System



Dialogue System (e.g. Siri, Alexa)

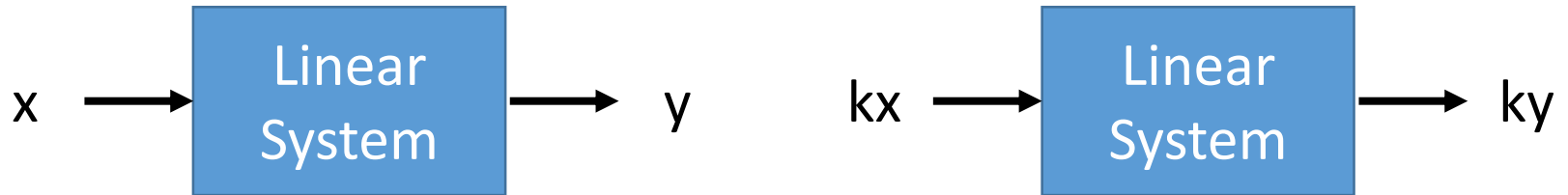


Communication System

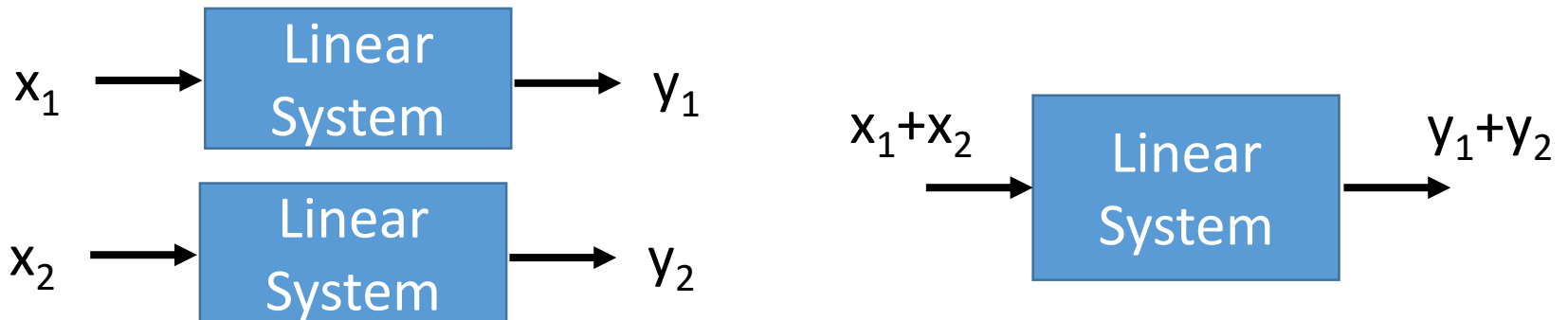


Linear System

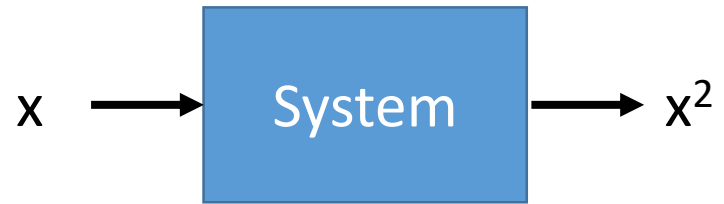
- Linear system have two properties
 - 1. Persevering Multiplication



- 2. Persevering Addition

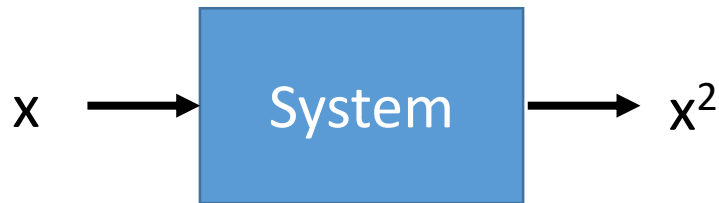


Linear?

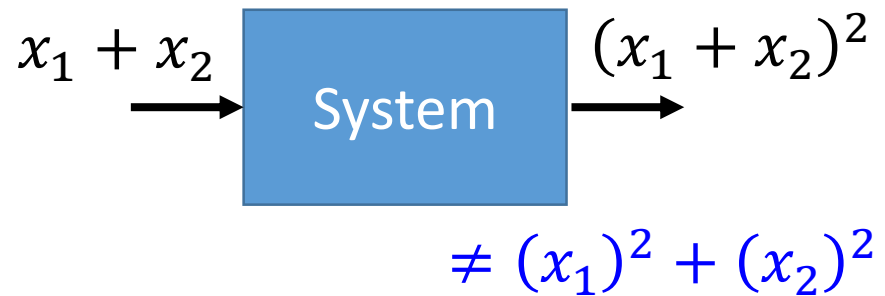
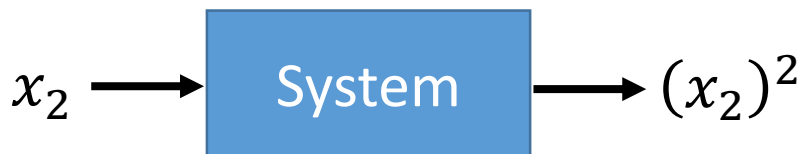


- Linear system have two properties

- 1. Persevering Multiplication

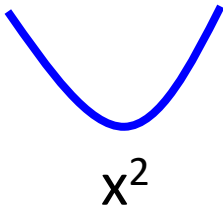


- 2. Persevering Addition



Linear?

- Derivative

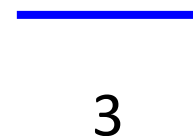
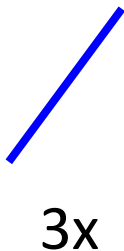
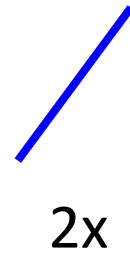


$$f \rightarrow f'$$

$$g \rightarrow g'$$

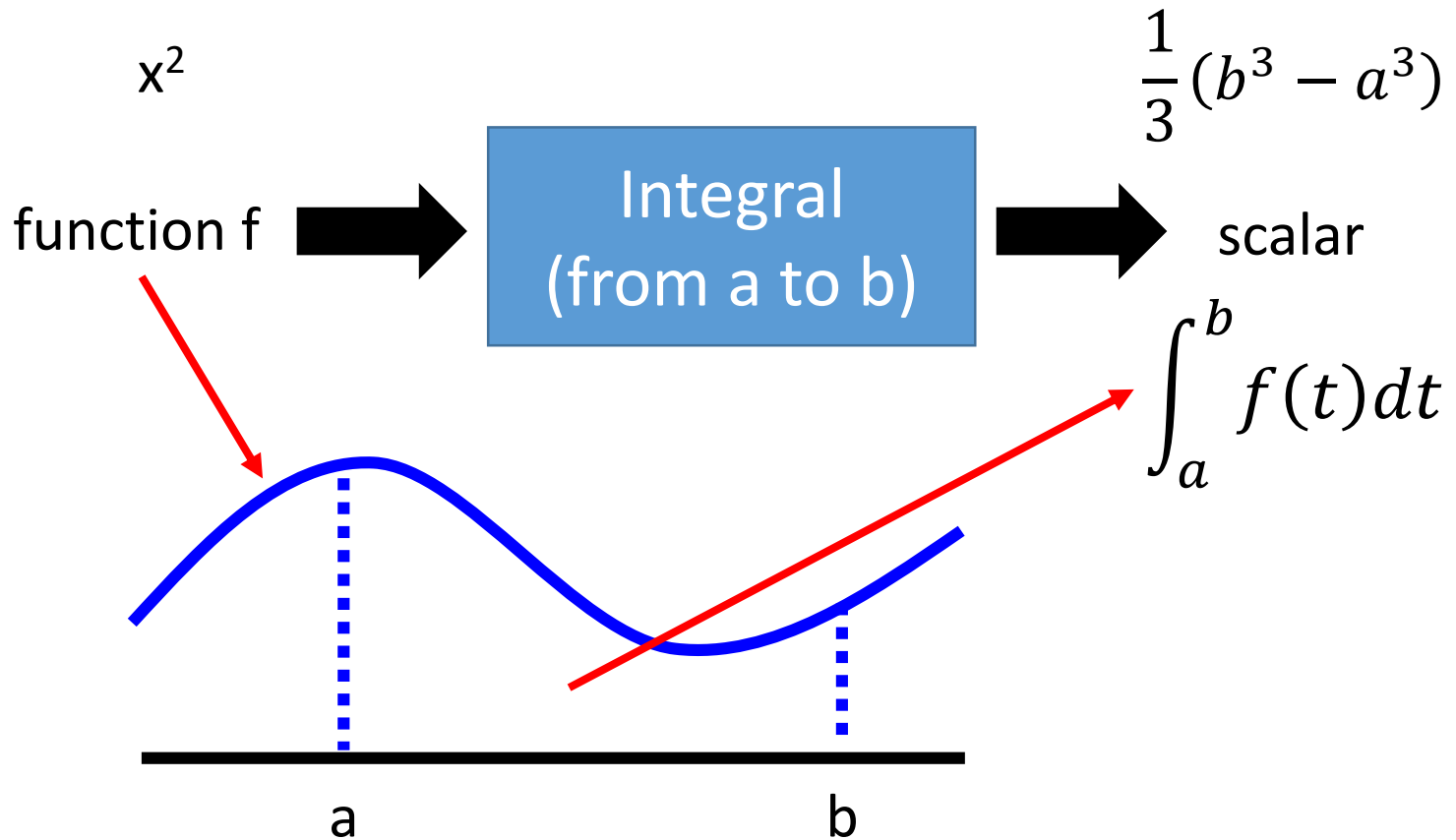
$$kf \rightarrow kf'$$

$$f + g \rightarrow f' + g'$$

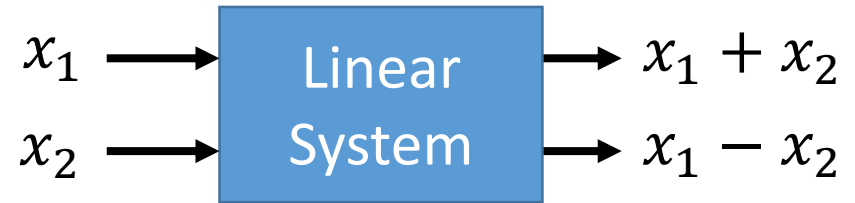


Linear?

- Integral from a to b

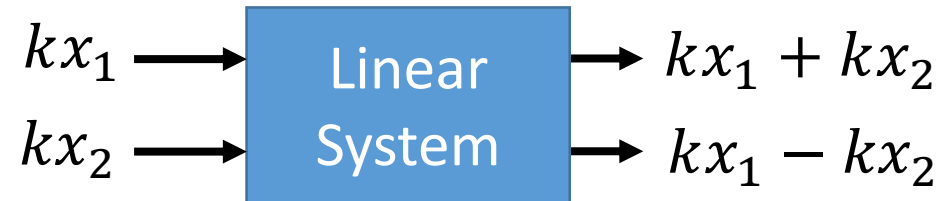
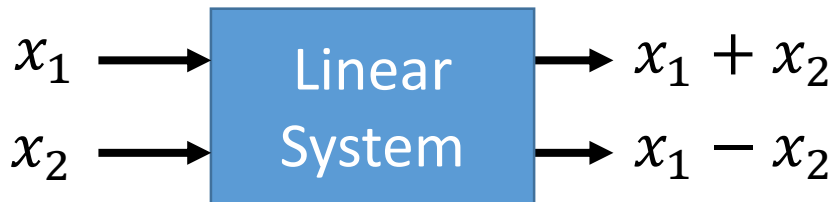


Linear System

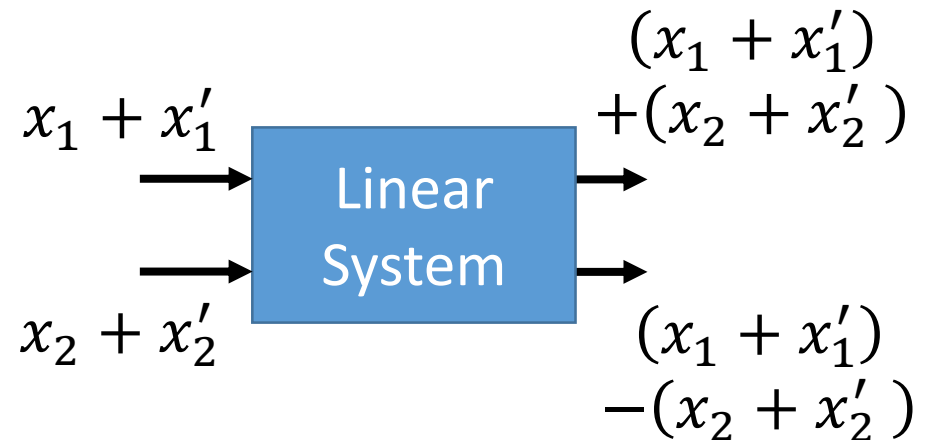
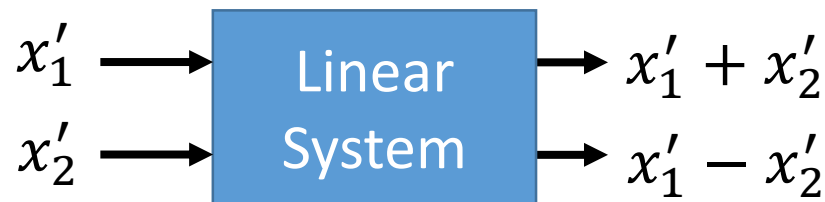
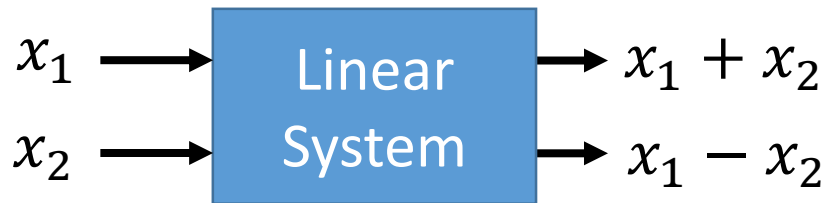


- Two properties

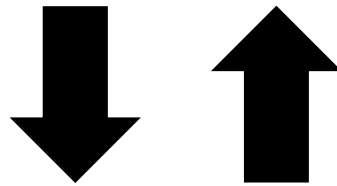
- 1. Persevering Multiplication



- 2. Persevering Addition



Linear System v.s. System of Linear Equations



$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

System of Linear Equations

多元一次聯立方程式

- I believe you know it.

The diagram shows a system of m equations with n variables. The equations are:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

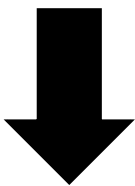
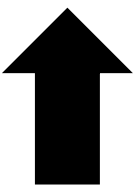
Annotations:

- An arrow points from the word "coefficient" to the a_{ij} terms.
- An arrow points from the word "n variables" to the x_1, x_2, \dots, x_n terms.
- A bracket on the left groups the equations and is labeled "m equations".

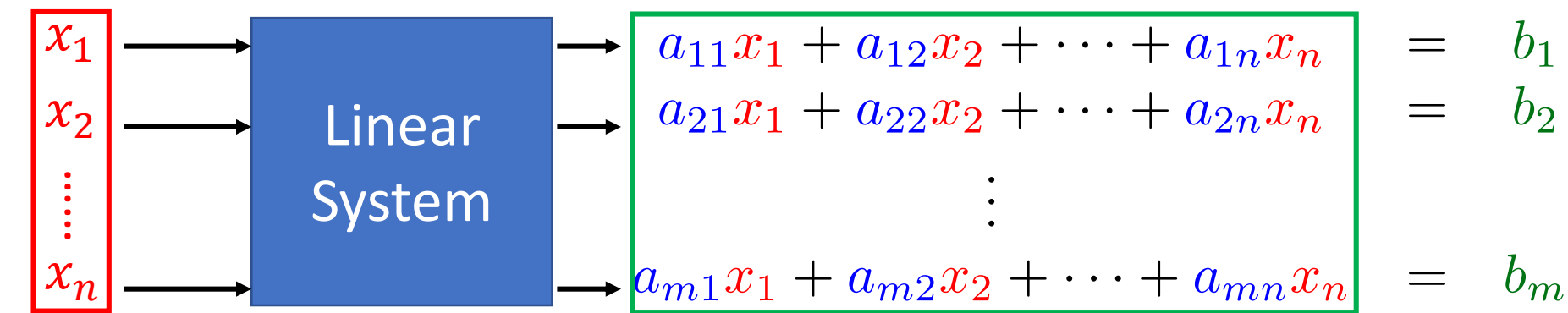
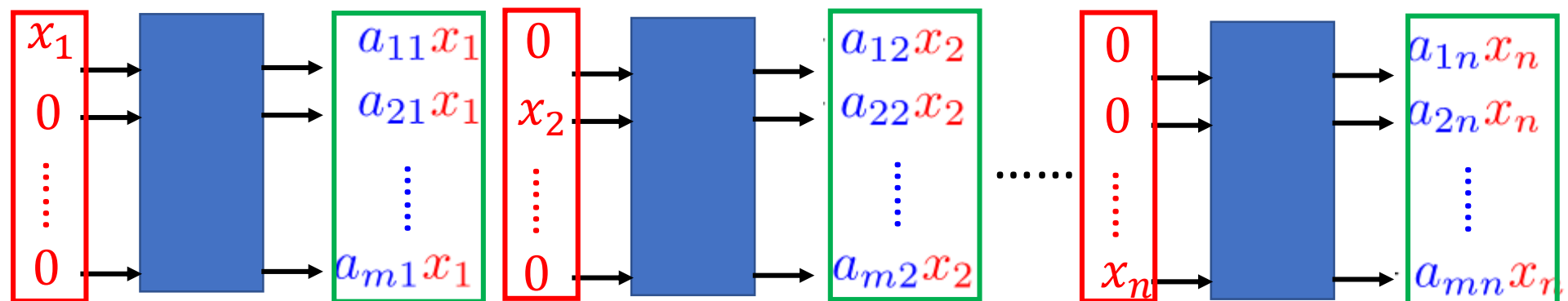
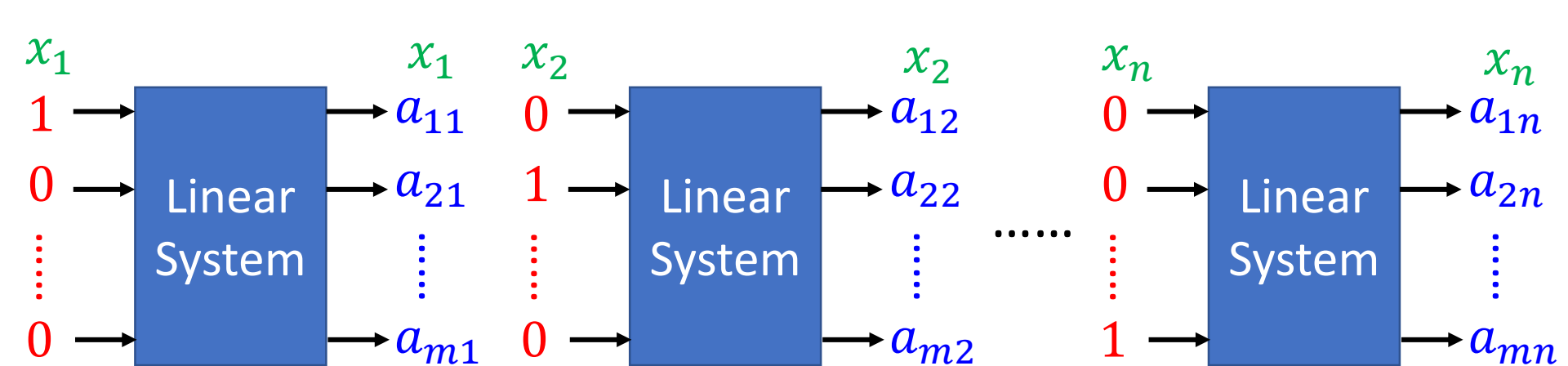
In this course, m and n can be large

Linear System v.s. System of Linear Equations



?   trivial

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

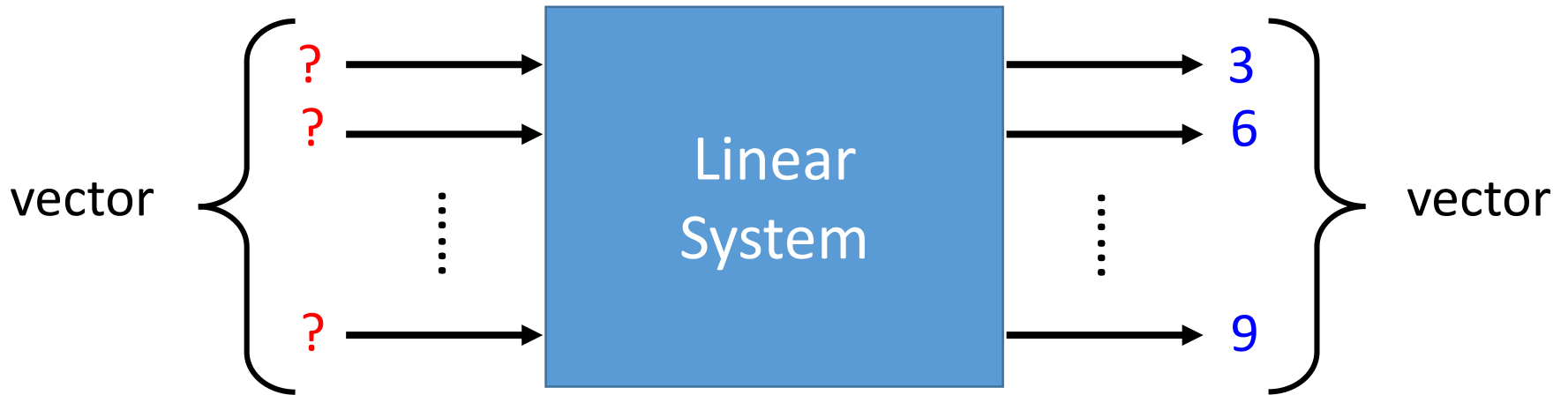


A linear system is described by a system of linear equations

What are we going to learn?

Chapter 1, Chapter 2, Chapter 3

Beyond vector
in Chapter 6



Does it have
solution?

Does it have
unique solution?

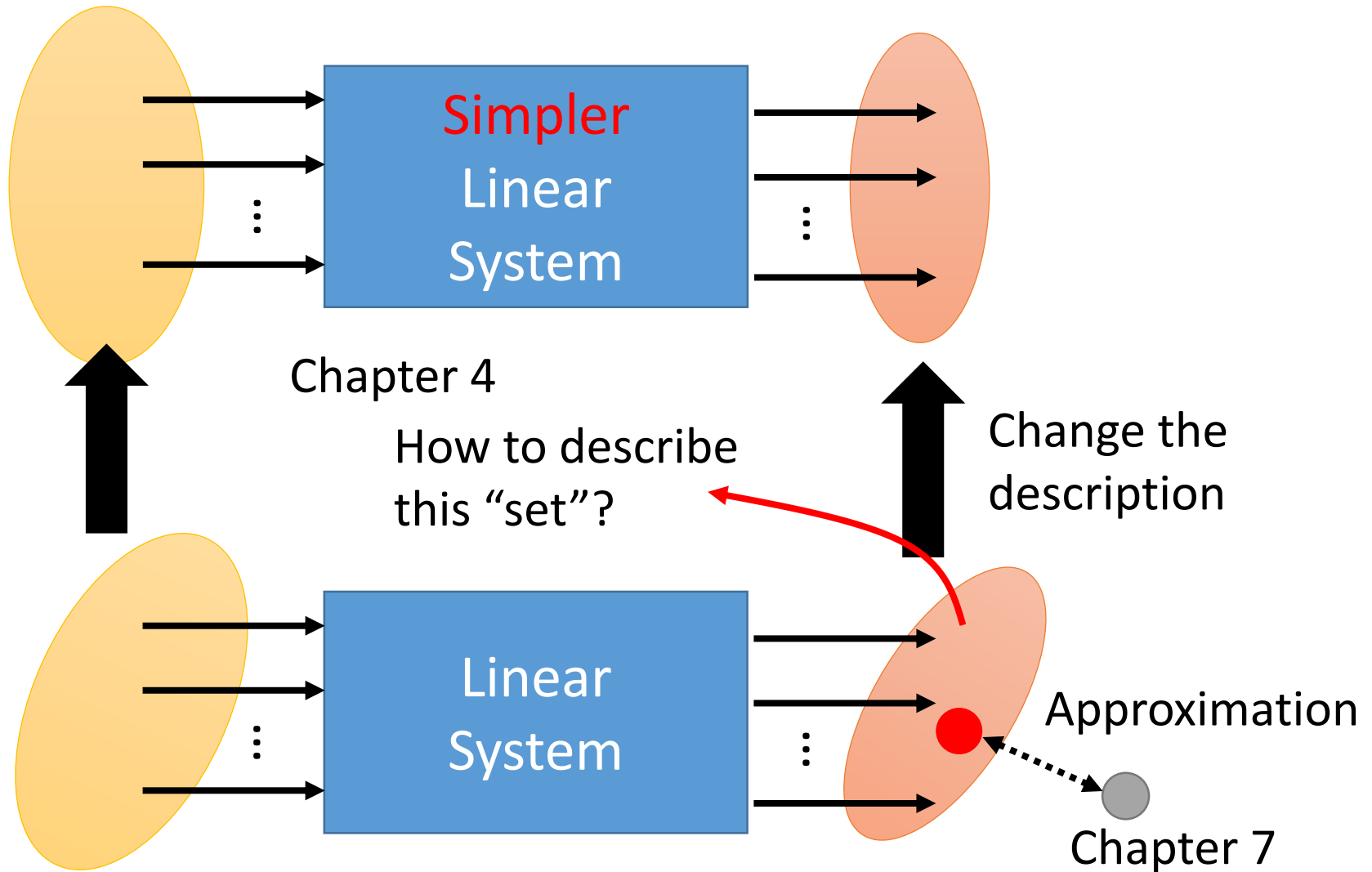
How to find
the solution?

Determinants
(行列式)

Beyond 3 X 3

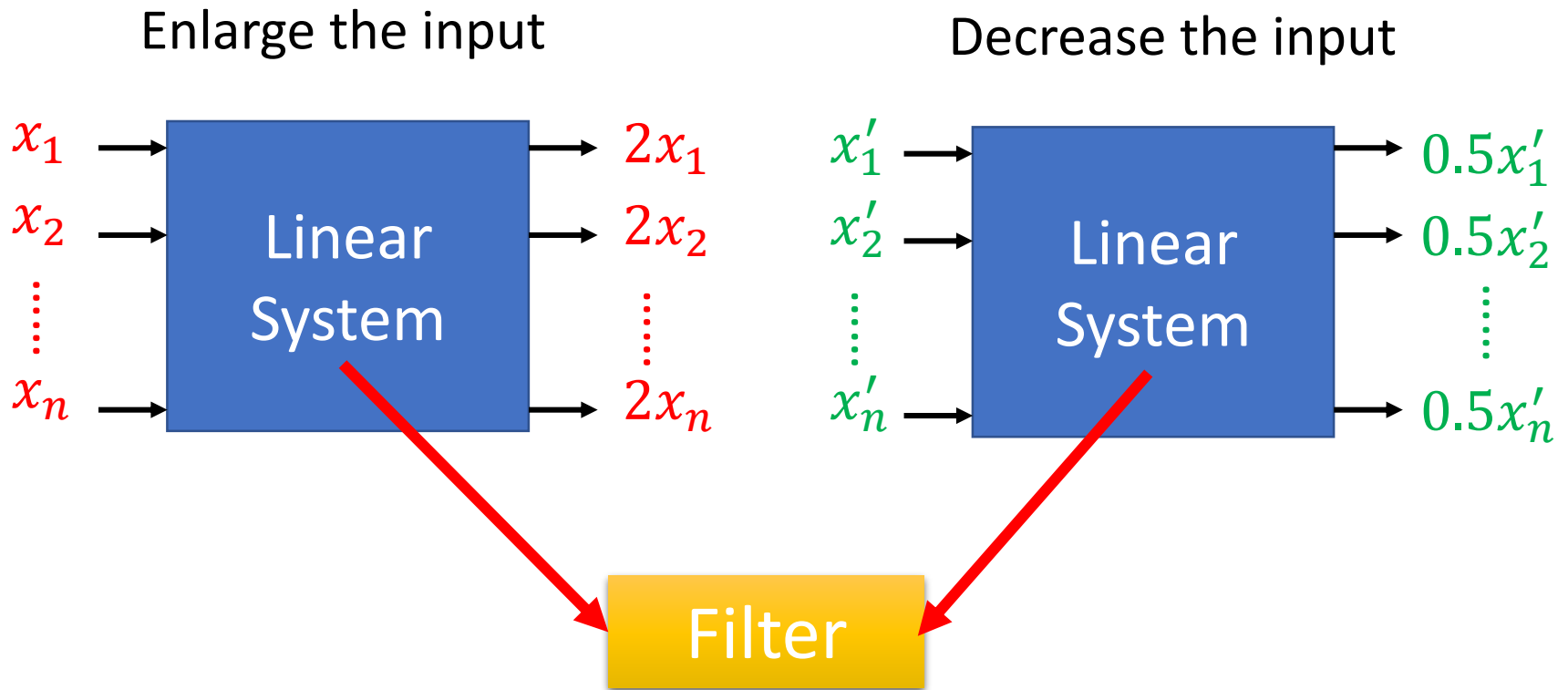
Different view from high school

What are we going to learn?



What are we going to learn?

- Chapter 5: EigenXXX



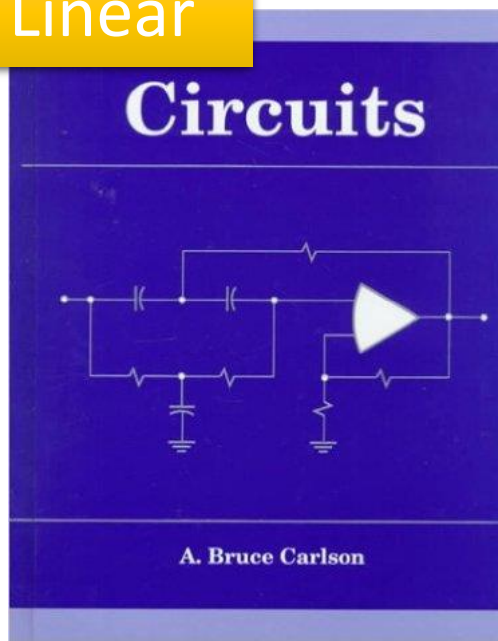


Application

Applications

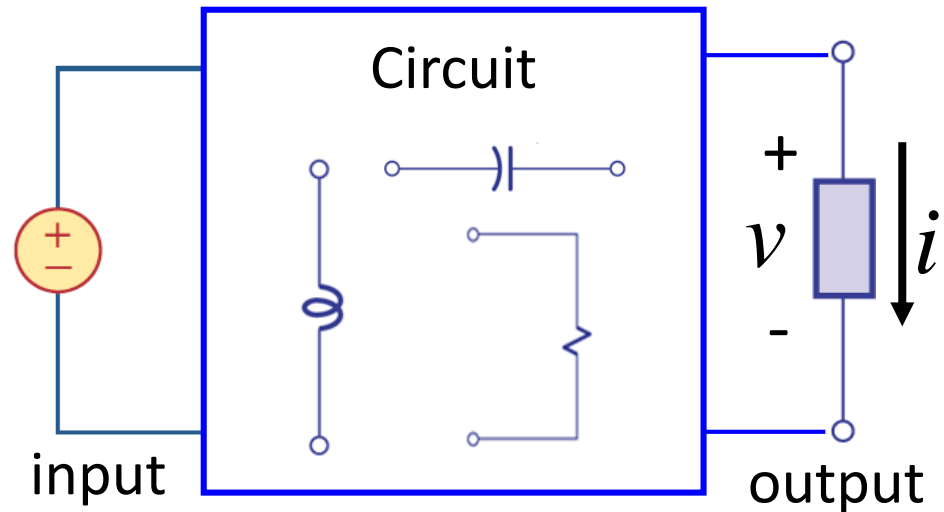
Most system are (assumed to) be linear

Linear



(大一必修)

Input: voltage source, current source
output: voltage and current on the load (燈泡、引擎)

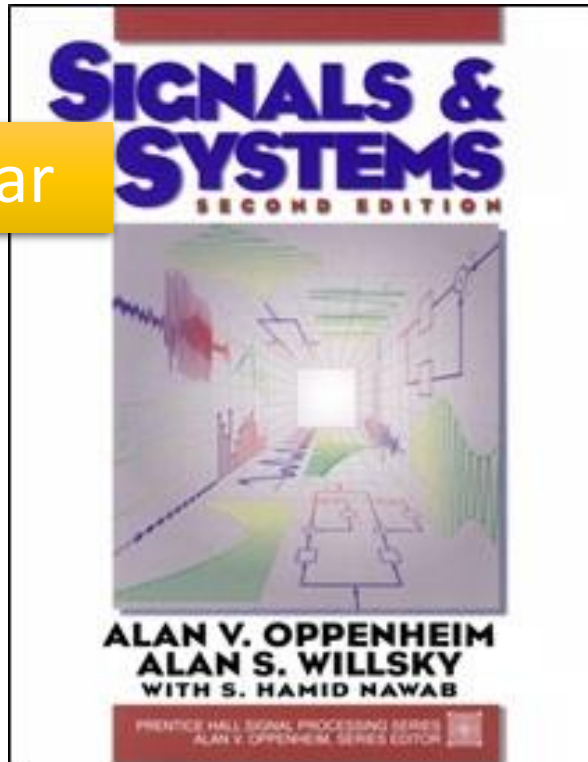


Linear System

Applications

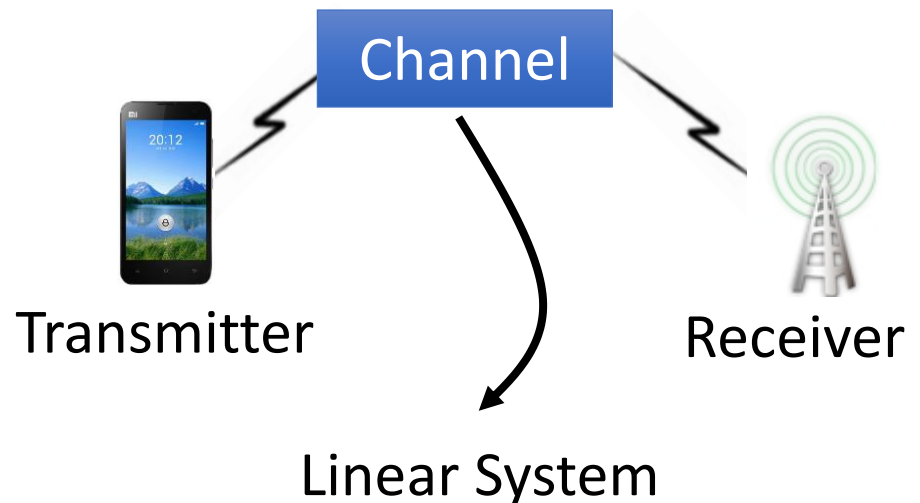
Most system are (assumed to) be linear

Linear

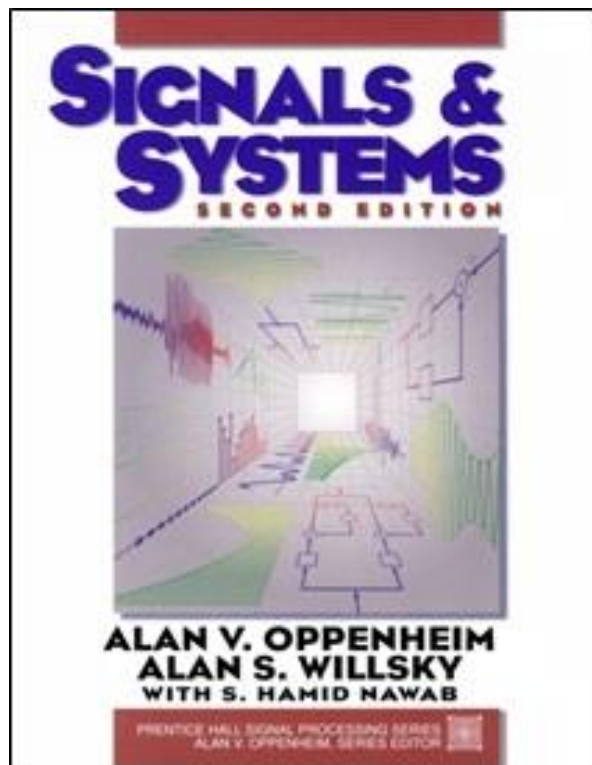


(大二必修)

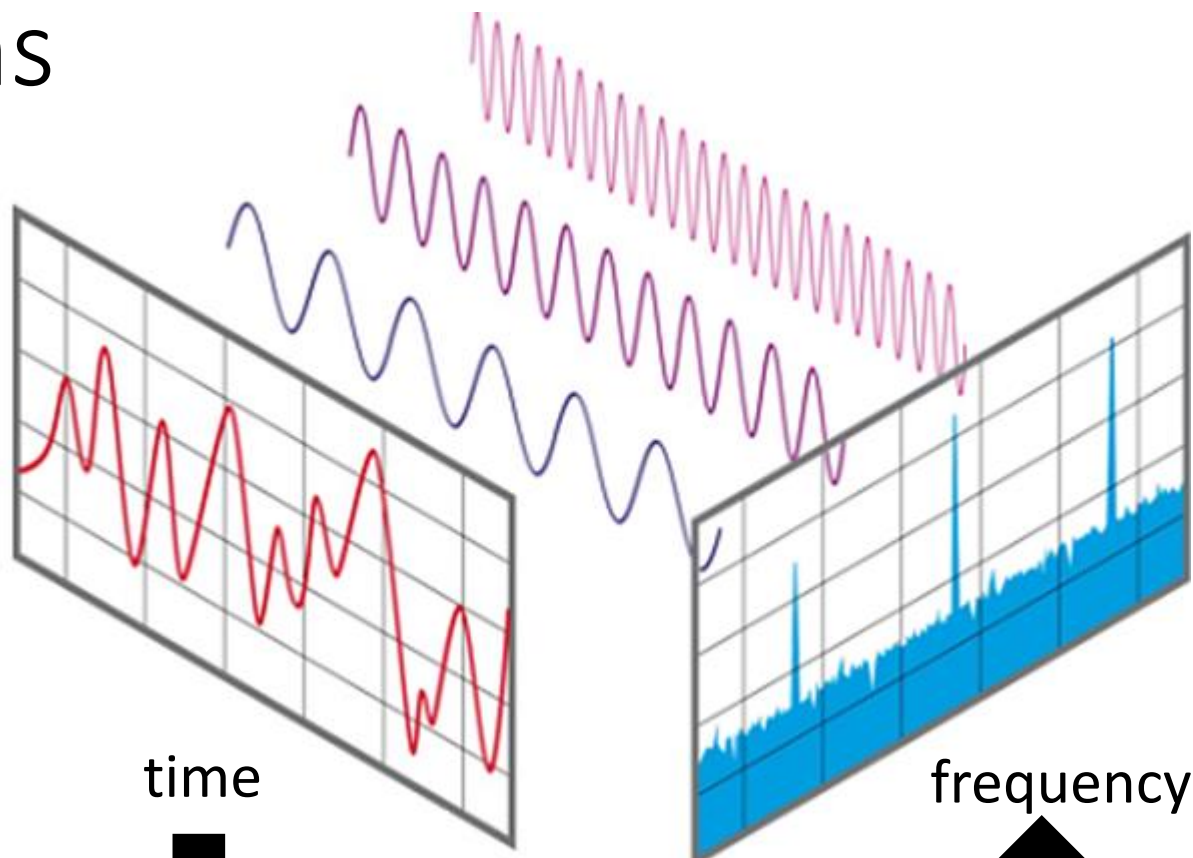
Communication System



Applications



(大二必修)



time

frequency

Fourier
Transform

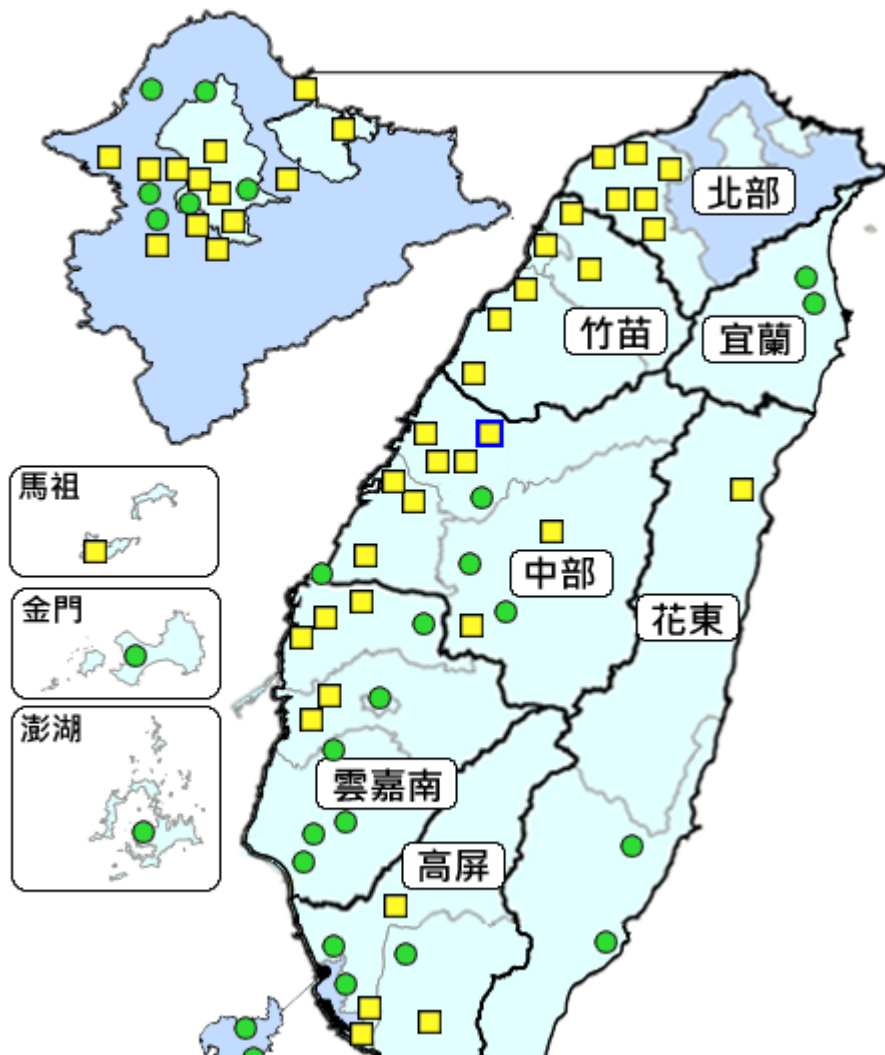
Linear System (Transform)

Applications

預測豐原站在下一個小時
會觀測到的 PM2.5

地區： >

發布時間：2017-09-29 08:00:00



豐原 (一般站)

AQI **64**
空氣品質指標 **普通**

O ₃ (ppb) 臭氧	8小時 移動平均	17
	小時 濃度	14
◎ PM _{2.5} ($\mu\text{g}/\text{m}^3$) 細懸浮微粒	移動 平均	21
	小時 濃度	27
PM ₁₀ ($\mu\text{g}/\text{m}^3$) 懸浮微粒	移動 平均	35
	小時 濃度	44

Applications

y : A 年 B 月 C 日 N 時的 **PM2.5**

x_k : A 年 B 月 C 日 $N - k$ 時的 **PM2.5** 觀測值



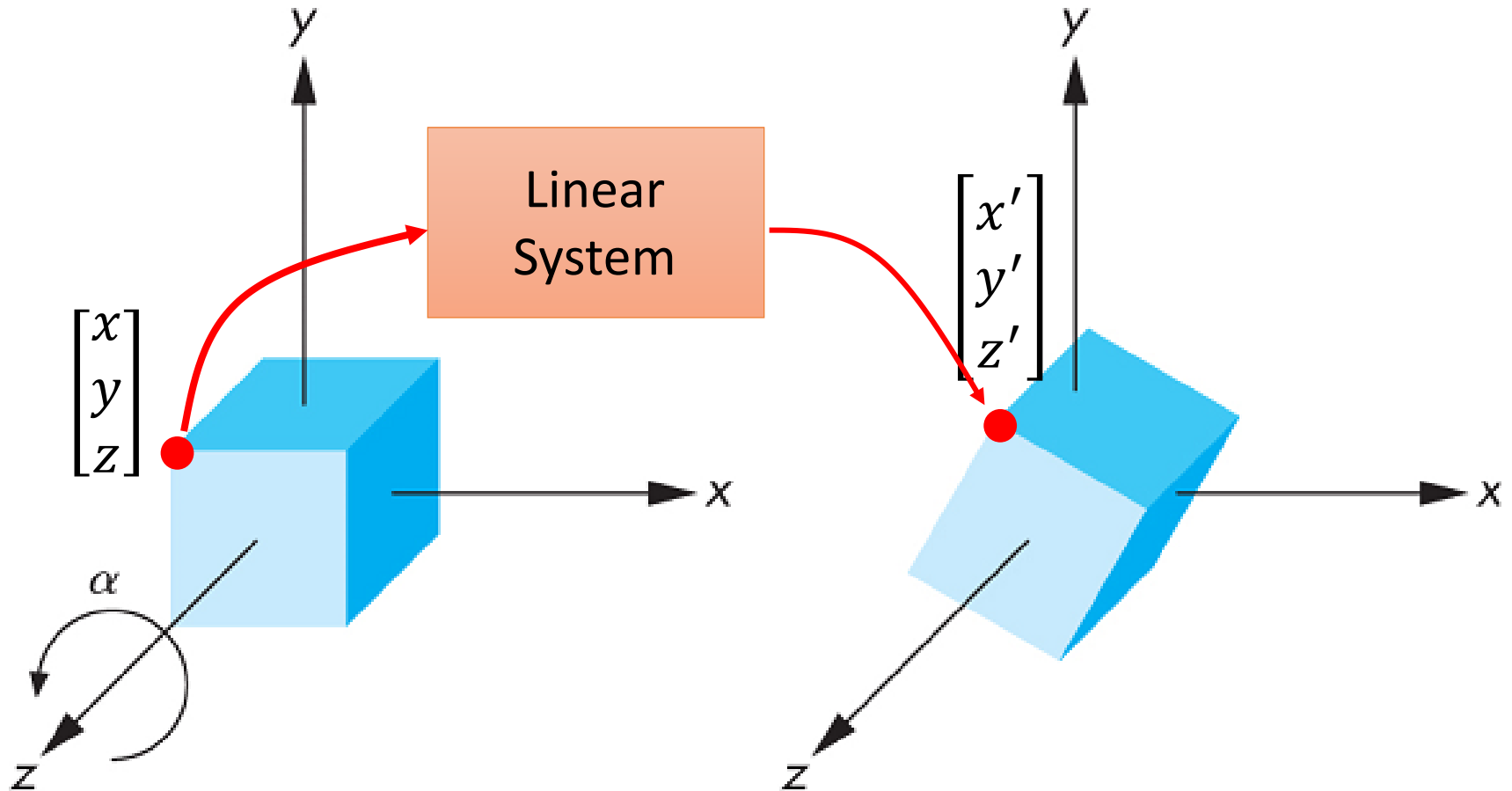
Linear system

$$y = w_1x_1 + w_2x_2 + w_3x_3$$

Where do w_1 , w_2 and w_3 come from?

Applications - Computer Graphics

Section 7.9 of Textbook



全部 地圖 圖片 新聞 影片 更多 ▾ 搜尋工具

Applications

P336 of Textbook

約有 5,160,000 項結果 (搜尋時間：1.01 秒)

國立臺灣大學

www.ntu.edu.tw/ ▾

包含學校簡介、系所介紹、校園資訊。成立於1928年，前身為臺北帝國大學。1945年更名為臺灣大學。

您已造訪這個網頁 3 次。上次造訪日期：2016/5/1

ntu.edu.tw 內容的搜尋結果



招生資訊

碩士班招生 - 學士班轉學考 - 碩士班甄試入學 - ...

資訊網路與多媒體研究所

碩士班 - 本所成員 - 碩士班修業規定 - 課程介紹 - ...

myNTU臺大人入口網

... 計中帳號登入！SSO1.3. 登入 - ※ 預防帳號遭盜用，請定期修改 ...

學術單位

文學院 - 工學院 - 理學院 - 生物資源暨農學院 - ...

圖書館

館藏資源 - 電子資源 - 資料庫 - 開放時間 - 學生 - ...

國立臺灣大學學士班轉學考試

招生名額及科目 一般生(不招收陸生)陸生(限在臺就讀). 預定日程 - ...

國立臺灣大學- 維基百科，自由的百科全书

<https://zh.wikipedia.org/zh-tw/國立臺灣大學> ▾

國立臺灣大學，簡稱臺灣大學、臺大，乃臺灣最早的現代綜合大學，前身是於1928年創立的臺北帝國大學，籌設之初定位為只辦醫學和農學的實業大學，伊澤多喜男力排 ...

國立臺灣大學National Taiwan University - Facebook

www.facebook.com ▾ Places ▾ Taipei, Taiwan ▾ Landmark ▾

★★★★ 評分：1.8 - 11,822 票

國立臺灣大學National Taiwan University, Taipei, Taiwan. 37715 likes · 1580 talking about this · 175196 were here. 國立臺灣大學粉絲專頁.



<http://incomebully.com/does-pr-pagerank-still-matter/>

Linear Algebra is Important



在電機系
如果沒有把線性代數學好
就好像沒有學念能力

卻到了天空鬥技場兩百樓

李宏毅